Chapter 12
Probability

12.1 Sample Spaces and Probability

12.2 Independent and Dependent Events

12.3 Two-Way Tables and Probability

12.4 Probability of Disjoint and Overlapping Events

12.5 Permutations and Combinations

12.6 Binomial Distributions

12.2 Independent and Dependent Events

Vocabulary

- **Independent Events** - Two (or more) events whose outcomes of one *does not* affect the other.
- **Dependent Events** - Two (or more) events whose outcomes *do* affect each other.

Independent or Dependent?

a) Rolling two dice.

b) Picking two numbered slips from a bag without putting any back.

12.2 Independent and Dependent Events

Probability of Independent Events

Two events A and B are independent events if and only if the probability that both events occur is the **product** of the probabilities of the events.

P(A and B) = P(A) • P(B)

Example

Rolling two 6-sided dice. What is the probability of rolling two sixes?

12.2 Independent and Dependent Events

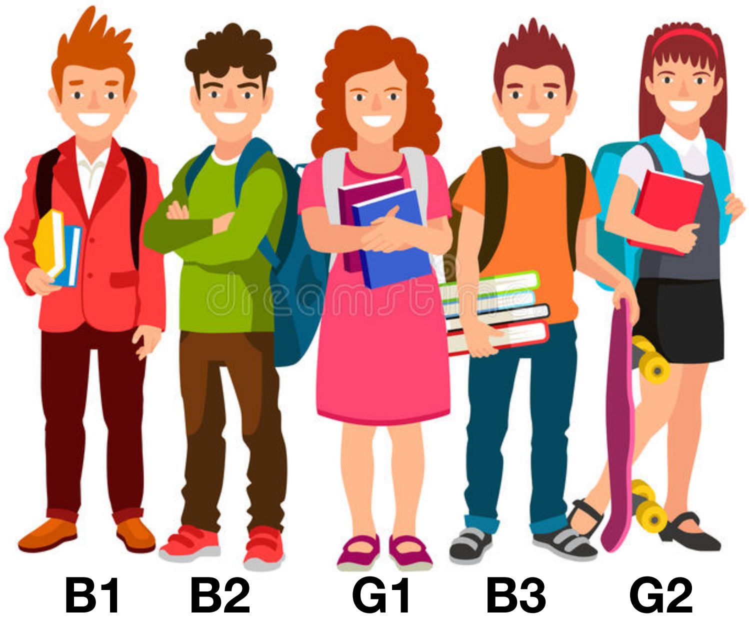
Example - Independent or Dependent?

- A group of five students include three boys and two girls. Mr Greenstein randomly selects one to be the speaker and a different student to be the recorder. Determine whether randomly selecting a boy first and randomly selecting a different boy second are independent.

Sample Set

Speaker/Recorder

B1,B2	B2,B1	B3,B1	G1,B1	G2,B1
B1,B3	B2,B3	B3,B2	G1,B2	G2,B2
B1,G1	B2,G1	B3,G1	G1,B3	G2,B3
B1,G2	B2,G2	B3,G2	G1,G2	G2,G1



12.2 Independent and Dependent Events

Conditional Probability

The probability that event B occurs given that event A has occurred is called the conditional probability of B given A.

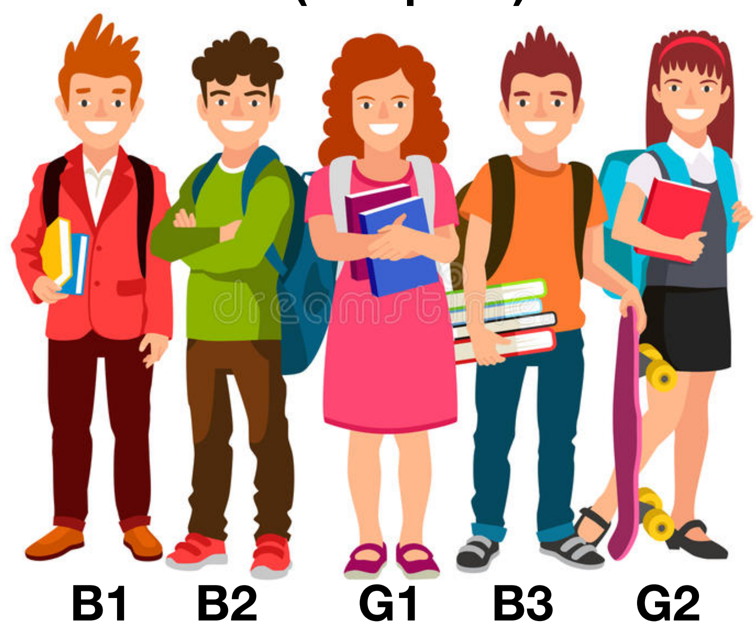
P(B | A)

Example: What is the probability of choosing G1 given you already chose G2 as speaker? In other words: **P(G1 | G2)**?

Sample Set

Speaker/Recorder

B1,B2	B2,B1	B3,B1	G1,B1	G2,B1
B1,B3	B2,B3	B3,B2	G1,B2	G2,B2
B1,G1	B2,G1	B3,G1	G1,B3	G2,B3
B1,G2	B2,G2	B3,G2	G1,G2	G2,G1



12.2 Independent and Dependent Events

Conditional Probability

Example: A quality-control inspector checks for defective parts. The table shows the results of the inspector's work. Find (a) the probability that a defective part "passes," and (b) the probability that a non-defective part "fails."

	Pass	Fail
Defective	3	36
Non-defective	450	11

a. $P(\text{pass} \mid \text{defective}) = \dfrac{\text{Number of defective parts "passed"}}{\text{Total number of defective parts}}$

$$= \frac{3}{3 + 36} = \frac{3}{39} = \frac{1}{13} \approx 0.077, \text{ or about } 7.7\%$$

b. $P(\text{fail} \mid \text{non-defective}) = \dfrac{\text{Number of non-defective parts "failed"}}{\text{Total number of non-defective parts}}$

$$= \frac{11}{450 + 11} = \frac{11}{461} \approx 0.024, \text{ or about } 2.4\%$$

12.2 Independent and Dependent Events

Probability of Dependent Events

If two events A and B are dependent events, then the probability that both events occur is the **product** of the probability of the **first event** and the **conditional probability of the second event** given the first event.

P(A and B) = P(A) • P(B|A)

Example

Picking two numbered slips randomly from a bag of numbered slips without putting any back.

a) What is the probability of choosing 2 and then 3?

b) What is the probability of choosing 1 or 4 and then 5?

12.2 Independent and Dependent Events

Revisiting Conditional Probability

Start with the probability of dependent events:

$$\boxed{P(A \text{ and } B) = P(A) \cdot P(B|A)}$$

Using algebra, divide each side by P(A).

$$\boxed{P(B|A) = \frac{P(A \text{ and } B)}{P(A)}}$$

Formula for Conditional Probability

Picking two numbered slips randomly from a bag of numbered slips without putting any back.

a) What is the probability of choosing 2 and then 3?

b) What is the probability of choosing 1 or 4 and then 5?

12.2 Independent and Dependent Events

Calculating Probability

Example

You randomly select 3 cards from a standard deck of 52 playing cards. What are the chances they are all hearts when:

a) you place the cards back into the deck before you choose again?
b) you do not place the cards back into the deck before choosing again?

Independent	**P(A and B) = P(A) • P(B)**
Dependent	**P(A and B) = P(A) • P(B|A)**